List of Participants and Abstracts

Asymptotic Properties and Operator Norms of Gauß-Weierstraß Operators and Their Left Quasi Interpolants

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The Gauß-Weierstraß convolution operators W_n $(n = 1, 2, 3, \ldots)$ are defined by

$$(W_n f)(x) = \sqrt{\frac{n}{\pi}} \int_{-\infty}^{\infty} f(t) \exp\left(-n(t-x)^2\right) dt \tag{1}$$

(see, e.g., [5, Section 5.2.9]). They are positive linear approximation operators which are applicable to the class $L_c(\mathbb{R})$ of all locally integrable real functions f on $\mathbb{R}$ satisfying the growth condition $f(t) = O\left(e^{ct^2}\right)$ as $t \to \pm\infty$, for some $c > 0$, provided that $n > c$. If f is continuous we have

$$\lim_{x \to \infty} (W_n f)(x) = f(x)$$

uniformly on compact subsets of $\mathbb{R}$.

The complete asymptotic expansion of $(W_n f)(x)$ as n tends to infinity appears to be a special case of the results [2] on a more general operator defined by Altomare and Milella [4].

Recently, Sablonnière [6] defined quasi-interpolants of W_n and studied their basic properties including the operator norm for bounded functions with respect to the sup-norm. In particular, he proved asymptotic expansions for polynomials and functions having a bounded higher order derivative on the whole line.

We present the complete asymptotic expansions of $(W_n f)(x)$ and the left quasi-interpolant $\left(W_n^{[r]} f\right)(x)$ as n tends to infinity, for functions belonging to the class $L_c(\mathbb{R})$ which are assumed to be only locally smooth.

The corresponding results for the Favard operators which are the discrete version of W_n can be found in [3, 1].

Finally, we consider the operator norms of W_n and $W_n^{[r]}$ when acting on various function spaces.

2010 Mathematics Subject Classification. 41A36, 41A60, 44A35.

Key words and phrases. Approximation by positive operators, convolution operators, asymptotic expansion.

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http://dx.doi.org/10.1016/j.jat.2013.12.003

Approximation of Functions by Genuine Bernstein-Durrmeyer Type Operators

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Very recently, in [1] Chen et. al introduced and considered a new generalization of Bernstein polynomials depending on the parameter α. As classical Bernstein operators, these operators also provide interpolation at the end points of $[0,1]$ and they have the linear precision property which means they reproduce the linear functions.

In this paper we introduce the genuine α-Bernstein-Durrmeyer operators. Some approximation results, which include local approximation, error estimation in terms of the modulus of continuity and weighted approximation are obtained. Also, the convergence of these operators to certain functions is shown by illustrative graphics using MAPLE algorithms.

2010 Mathematics Subject Classification. 41A36, 41A25.

Key words and phrases. *Positive linear operators, rate of convergence, genuine Bernstein-Durrmeyer operator.*

References

[1] X. Chen, J. Tan, Z. Liua, J. Xie,*Approximation of functions by a new family of generalized Bernstein operators*, J. Math. Anal. Appl., 450 (2017), 244-261.

On a Modification of Srivastava-Gupta Operators

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In this paper we consider a modification of the well known sequence of summation-integral operators, the Srivastava-Gupta operators, in order to achieve faster convergence for our operators over the classical ones. The order of approximation for our proposed operators via Peetre K-functional and weighted approximation properties are also studied. In the last section with the help of Maple, some numerical considerations regarding the approximation properties, are considered

2010 Mathematics Subject Classification. 41A10, 41A25, 41A36.

Key words and phrases. *Srivastava-Gupta operators, rate of convergence, linear positive operators.*

The Study of Some Generalizations Obtained from an Approximation Process

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Starting from a general class of linear positive operators of discrete type, we apply its various modifications to create new approximation processes of discrete or continuous type. Working in different functions spaces, our aim is to establish the transfer of approximation properties from the initial sequence to the new constructions. Particular cases are analyzed.

2010 Mathematics Subject Classification. 41A36, 41A25.

Key words and phrases. Linear and positive operator, weighted space, modulus of smoothness, Kantorovich operator.

The Method of Iterated Splines for Integral Equations with Vanishing Delay

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The method of iterated splines is presented for constructing the numerical solution of Volterra and Fredholm integral equations with vanishing delay. We study as particular cases two-point boundary value problems of even order and initial value problems with vanishing delay, including the well-known pantograph equation. The convergence of the method is proved by providing the error estimate. The numerical stability regarding the choice of the first iteration is investigated. The accuracy, the convergence and the numerical stability are illustrated by some numerical experiments.

*2010 Mathematics Subject Classification.*34K28, 45D05, 45B05.

Key words and phrases. Functional integral equations, vanishing delay, numerical approximation of solutions, iterated splines, numerical stability.

Optimal Criteria for Fitting Splines

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An optimal algorithm working on iterated steps is applied for smooth interpolation of given data regarding optimal criteria. For several specified types of quadratic and cubic splines an objective function is minimized selecting on each subinterval the best spline type polynomial with respect to the optimal requirement. The obtained spline by connecting these polynomials preserves both the required optimality and the C1 smoothness property.

2010 Mathematics Subject Classification. 65S05.

Key words and phrases. Optimal criteria, C1 smooth cubic splines, not-a-knot quadratic spline.

Quantitative Results for the Iterates of Some Positive Linear Operators

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Abstract: In this article we obtain rates of the convergence for the iterates of some positive linear operators which preserve certain functions. We use the first and the second order modulus of smoothness. Some examples are given

2010 Mathematics Subject Classification. 41A36, 41A25.

Key words and phrases. Positive linear operators, iterates, moduli of smoothness.

B-Spline Curve Approximation and its Application in Medical Image Data

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In our days, approximation methods have been integrated into social practice using computer through the study of various phenomena that occur in numerical analysis problems. Based on the theory of approximation, this article shows that for all modalities, spline interpolation constitutes the best trade-off accuracy and shape fidelity needed for medical operations. In these applications, methods of image interpolation can be used from which the spline interpolation is the most widespread. It has been shown that B-spline interpolation offers superior cost-performance tradeoff over other medical interpolation algorithms.

2010 Mathematics Subject Classification. 41A15, 41A05, 92C50.

Key words and phrases. B-splines, interpolation, medical applications.

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On Some Common Generalizations of Two Classes of Integral Inequalities for Twice Differentiable Functions

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In this paper we establish a general integral identity for twice differentiable functions via fractional calculus. Then, we deduce some common generalizations of two classes of integral inequalities of perturbed trapezoidal and Bullen type for twice differentiable functions involving Riemann-Liouville fractional integrals.

2010 Mathematics Subject Classification. 26D15, 26A33, 26A51.

Key words and phrases. Differentiable functions, integral inequalities, Riemann-Liouville fractional integrals.

Some Applications Obtained from the Lipschitz Continuity of Solutions of Quadratic Programs

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Recently, I proved the Lipschitz continuity, with sharpest Lipschitz constant, of the solution function of general quadratic programs as function in two parameters, one in the quadratic function and the other in the right hand-side of the polyhedral constraints. In connection with this result, I will discuss two applications. First, I will consider a best approximation problem in the L[0,1] space, for which the exact analytical solution cannot be obtained. Instead, transforming this problem into a general quadratic program, by using the aforementioned Lipschitz continuity property, we can approach the exact solution of the problem with an arbitrary precision. The second problem, discuss a typical approximation problem in fuzzy analysis which depends on reasonably high number of parameters. We will find an explicit Lipschitz constant for the solution as function of these parameters.

2010 Mathematics Subject Classification. 90C20, 41A50.

Key words and phrases. Lipschitz continuity, quadratic program, best approximation, polyhedron.

Quantitative Estimates in Approximation by Choquet Type Integral Operators

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Qualitative results and quantitative estimates in terms of the modulus of continuity for the approximation by the multivariate non-linear Bernstein-Durrmeyer operator, written in terms of the Choquet integral (called as Bernstein-Durrmeyer-Choquet operator) with respect to a monotone and submodular set functions μ on the standard d-dimensional simplex were obtained by the authors in two previous papers. In the present talk, for the univariate Bernstein-Kantorovich-Choquet, Szász-Mirakjan-Kantorovich-Choquet, Baskakov-Kantorovich-Choquet operators and for the Picard-Choquet operators, Gauss-Weierstrass-Choquet operators and Poisson-Cauchy-Choquet operators, we present similar quantitative approximation estimates, uniform and pointwise in terms of the modulus of continuity and in some cases, in the L^p-norm in terms of a K-functional.

*2010 Mathematics Subject Classification.*41A35, 41A25, 28A12, 28A25.

Key words and phrases. Monotone and submodular set functions, distorted Lebesgue measures, Choquet integral, Bernstein-Kantorovich-Choquet operator, Szász-Kantorovich-Choquet operator, Baskakov-Kantorovich-Choquet operator.

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On a Class of Stochastic Optimization Problem

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We present results related to stochastic optimization problems with mixed expectation and per scenario constraints. We analyze numerical schemes for solving such problems together with convergence and computational results.

2010 Mathematics Subject Classification. 90C15.

Key words and phrases. Stochastic optimization problems, mixed expectation and per scenario constraints, sample average approximation.

On Positive Linear Operators with Equidistant Nodes Having Uniform Jackson Order of Approximation

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A sequence on positive linear operators with equidistant nodes are considered. It is shown that this sequences has uniform Jackson order of approximation and preservs monotonicity and convexity. Estimates in terms of Ditzian-Totik modulus of smoothness are given.

2010 Mathematics Subject Classification. 41A36, 41A17, 42A16.

Key words and phrases. Linear positive operators.Jackson type operators.Monotonicity and convexity.

Spectral Collocation for Problems on Unbounded Domains

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The aim of this communication is to argue that spectral collocation based on Laguerre, Hermite as well as Sinc functions offers reliable and accurate solutions to a large class of boundary value problems on unbounded domains.

We consider in turn non-standard eigenvalue problems, singular and/or nonlinear o. d. e. and p. d. e. ([2]). In order to estimate the accuracy of outcomes we display the behavior (the decreasing way) of the expansion coefficients of solutions. Because this method works in physical space, to get these coefficients in the spectral space we make use of the FFT or another polynomial transforms ([1]).

The functional background for the above method is provided by the important paper [3].

2010 Mathematics Subject Classification. 65L10, 65L15, 65L60, 65M70.

Key words and phrases. Unbounded domain, spectral collocation, eigenvalue problem, boundary value problem.

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On Decomposing the Bernstein Operator

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We discuss several aspects in regard to decomposing the classical Bernstein operator. Piecewise linear interpolation at equidistant points and a classical Beta-type operator are in the focus.

2010 Mathematics Subject Classification. 41A35, 41A05.

Key words and phrases. Decomposing Bernstein operator, piecewise linear interpolation, Beta operators.

A Nice Representation for a Link Between Bernstein-Durrmeyer and Kantorovich Operators

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In [1] Păltănea introduced a class of operators $B_{n,\rho}$ depending on a nonnegative real parameter ρ which constitute a nontrivial link between the genuine Bernstein-Durrmeyer operator and the classical Bernstein operator. We denote by $\mathcal{L}_B[0,1]$ the space of bounded Lebesgue integrable functions on $[0,1]$.

For $j \in \mathbb{N}_0$, $0 \le j \le n$, the Bernstein basis functions are given by

$$p_{n,j}(x) = \binom{n}{j} x^j (1-x)^{n-j},\ 0 \le j \le n,\ x \in [0,1].$$

Moreover, for $1 \le j \le n-1$,

$$\mu_{n,j,\rho}(t) = \frac{t^{j\rho-1}(1-t)^{(n-j)\rho-1}}{B(j\rho,(n-j)\rho)}$$

with Euler's Beta function $B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$, $x,y > 0$. For $n \in \mathbb{N}$, $n \ge 2$, $\rho \in \mathbb{R}_+$, Păltănea [1, Definition 2.1] defined the operators $B_{n,\rho} : \mathcal{L}_B[0,1] \longrightarrow \mathcal{P}_n$ by

$$\begin{aligned} B_{n,\rho}(f;x) &= p_{n,0}(x)f(0) + p_{n,n}(x)f(1) \\ &\quad + \sum_{j=1}^{n-1} p_{n,j}(x) \int_0^1 \mu_{n,j,\rho}(t)f(t)dt. \end{aligned}$$

In [2, Theorem 2.3] Gonska and Păltănea proved the convergence of the operators $B_{n,\rho}$ to the classical Bernstein operator $B_{n,\infty}$, i.e., they proved that for every $f \in C[0,1]$

$$\lim_{\rho\to\infty} B_{n,\rho}f = B_{n,\infty}f \text{ uniformly on } [0,1].$$

In our talk we consider the k-th order Kantorovich modifications, i.e., $B_{n,\rho}^{(k)} = D^k \circ B_{n,\rho} \circ I_k$, where D^k denotes the ordinary differential operator of order k and I_k the corresponding k-th order antiderivative. For $k=1$ this means that we look at a link between Bernstein-Durrmeyer and Kantorovich operators. We will show how these linking operators can be represented in terms of well-known classical operators.

2010 Mathematics Subject Classification. 41A36, 41A28, 41A10.

Key words and phrases. Linking operators, Bernstein-Durrmeyer operators, Kantorovich operators, k-th order Kantorovich modification.

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A Thunsdorff Type Inequality

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Thunsdorff's inequality was generalized in several ways : see, e.g., [1], [2] and the references therein. In this talk we extend the Thunsdorff type inequality established in [3] and discuss it in relation with a complementary inequality.

2010 Mathematics Subject Classification. 26D15.

Key words and phrases. Thunsdorff inequality, Chebyshev inequality, power mean inequality.

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About Some Inequalities in an Inner Product Space

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The aim of this presentation is to discuss new results concerning the Cauchy - Schwarz inequality in an inner product space and several applications.

2010 Mathematics Subject Classification. Primary 46C05; Secondary 26D15, 26D10.

Key words and phrases. Inner product space, Cauchy - Schwarz inequality.

About a Family of Bernstein Operators

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We generalize a class of Bernstein-type operators introduced recently in [1].

2010 Mathematics Subject Classification. 26D15, 26D20, 26D99, 41A36, 41A25.

Key words and phrases. Bernstein operators, convexity, degree of approximation.

References

[1] X. Chen, J. Tan, Z. Liu, J. Xie, Approximation of functions by a new family of generalized Berstein operators, *J. Math. Anal. Appl.*, 450 (2017) 244-261.

New Weighted Estimates with Moduli of Continuity

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We discuss certain types of estimates with second order weighted moduli of continuity, like:

$\omega_2^{\varphi,\lambda}(f,h) = \sup\{|f(x) + f(y) - 2f((x+y)/2)|,\ x, y \in I,\ |y - x| \leq 2h\psi^\lambda((x+y)/2)\}$,

where $\psi(x) = x(1-x)$, $0 \leq \lambda \leq 1/2$..

2010 Mathematics Subject Classification. 41A36, 41A25.

Key words and phrases. Moduli of continuity, positive linear operators.

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Applications in Scalar Optimization

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A scalar optimization problem is considered. We attach to it the first order approximated optimization problem and also the dual one. The idea is to characterize the optimal solutions and the saddle points for the Lagrangian of the scalar optimization problem and obtain new results that characterize also the first order approximated optimization problem and the dual optimization problem. We present examples and results for these problems related to the optimal solutions and saddle points for the Lagrangian.

2010 Mathematics Subject Classification. Primary 90C26, Secondary 90C30, 90C46

Key words and phrases. Optimal solution, saddle point, optimization problem, dual of problem (P), $(0,1)-\eta-$ approximated optimization problem.

About an Inverse Form of Bernstein Operators

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In 1989, Paul Sablonniére introduced left Bernstein quasi-interpolant operator

$$B_n^{(K)}f=\sum_{k=0}^{K}\alpha_k^n\left(B_nf\right)^{(k)},\quad f:[0,1]\to\mathbb{R},\quad x\in[0,1]$$

and proved that the sequence of the approximating polynomials converges pointwise in high-order rate to each sufficiently smooth approximated function. The idea of studying this operators used as starting point the fact that, a polynomial of degree $\leq n$ interpolating the coefficients of a polynomial g, can be expressed as a linear combination of derivatives of g.

In 2000, M.S. Floater and T. Lyche also consider a polynomial of degree $\leq n$, $A_n(g)$, passing through the control points and interpolating the Bernstein-Bezier coefficients:

$$A_n(g)=\sum_{k=0}^{d}\frac{(n-k)!}{n!}\cdot S_{n,k}(x)\cdot\frac{g^{(k)}(x)}{k!},\quad x\in\mathbb{R},$$

where

$$S_{n,k}(x)=\sum_{i=0}^{k}s_{n,k,i}(x)$$

and

$$s_{n,k,i}(x)=(-1)^{k-i}\binom{k}{i}x^{k-i}\frac{(n-i)!}{(n-k)!}nx(nx-1)...(nx-i+1)$$

for $g \in \Pi_d$ and $d \leq n$. They showed that $A_n(g)$ can be viewed as an inverse to the Bernstein polynomial operator.

Following the above ideas, in the present paper we will construct two representations for Inverse Bernstein operators and discuss about asymptotic convergence.

2010 Mathematics Subject Classification. 41A10, 41A25, 41A35, 65D05.

Key words and phrases. Approximation by polynomials, Bernstein operators, inverse Bernstein operators.

The Index of Coincidence for the Binomial Distribution is Log-Convex

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Properties like convexity, log-convexity, complete monotonicity of entropies and indices of coincidence have been investigated in recent papers: see, e.g., [1-4] and the references therein.

In this talk we are concerned with the index of coincidence for the binomial distribution, defined by

$$F_n(x) := \sum_{k=0}^{n} \left(\binom{n}{k} x^k (1-x)^{n-k} \right)^2, \quad x \in [0,1].$$

Proving a conjecture from [3], we show that F_n is log-convex. The proof is based on

(i) the relationship of F_n with the classical Legendre polynomials (see [2]);

(ii) the property of F_n of being a Heun function (see [3]).

As applications, we derive properties of the Rényi and Tsallis entropies.

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Key words and phrases. Log-convex functions, entropies, Heun functions, Legendre polynomials.

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Convergence Properties of Certain Positive Linear Operators

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In the last year there is an increasing interest in modifying linear operators so that the new versions present a better degree of approximation than the original ones. In order to make the convergence faster, King [6] proposed for the classical Bernstein operators a new version that preserves the test functions e_0 and e_2 and presents a degree of approximation at least as good. Cárdenas-Morales et al. [3] extended this result considering a family of sequences of operators that preserve e_0 and $e_2+\alpha e_1$ with $\alpha \in [0,\infty)$. Gonska et al [5] studied a sequence of operators based on a continuous and strictly increasing function τ, defined on $[0,1]$, with $\tau(0)=0$ and $\tau(1)=1$. Note that these operators preserve e_0 and τ. Using the same type of technique or new methods many authors published new results dealing with this matter. In [4], the authors consider the sequence of linear Bernstein-type operators and in [1] are studied the modified Bernstein-Durrmeyer operators. Also, the modified Szasz operators were considered recently in [2].

In this presentation we will apply the method of τ function on a sequence $(L_n)_{n\geq 1}$ of linear positive operators, $L_n : C[0,1] \to \mathbb{R}$, defined as follows

$$L_n(f;x) = \sum_{k=0}^{n} p_{n,k}(x) f(x_{n,k}),\ x \in [0,1],$$

where every functions $p_{n,k}$ is continuous, $\sum\limits_{k=0}^{\infty} p_{n,k}(x) = 1$, and $0 = x_{n,0} < x_{n,1} < \cdots < x_{n,n} = 1$. A direct approximation theorem by means of the Ditzian-Totik modulus of smoothness and a Voronovskaja type theorem for the modified operators are obtained. Applications for some known positive linear operators are given.

2010 Mathematics Subject Classification. 41A36, 41A25.

Key words and phrases. Positive linear operators, rate of convergence, Ditzian-Totik modulus of smoothness, Voronovskaja type theorem.

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Convergence of Generalized (p,q)- Bernstein Polynomials

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Let $f \in C[0,1]$, $p, q \in (0,1)$, $0 < q < p \leq 1$ and $B_n(f,p,q;x)$ be generalized (p,q) Bernstein polynomials based in (p,q)-integers. These polynomials were introduced by M. Mursaleen, Khursheed J. Ansari and Asif Khan in 2015. We study convergence properties of the sequence $\{B_n(f,p,q;x)\}_{n=1}^{\infty}$. It is shown that in general these properties are quite different from those in the classical case $p = q = 1$.

2010 Mathematics Subject Classification. 41A36, 41A10.

Key words and phrases. Generalized Bernstein operators, (p, q) calculus.

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